Design of Experiments

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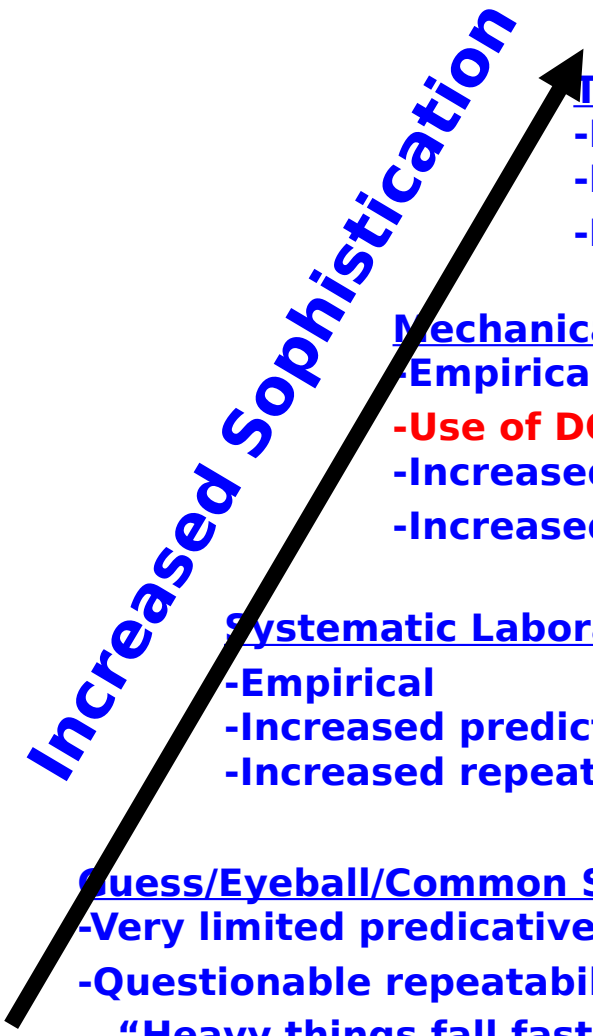
Understanding Manufacturing Processes



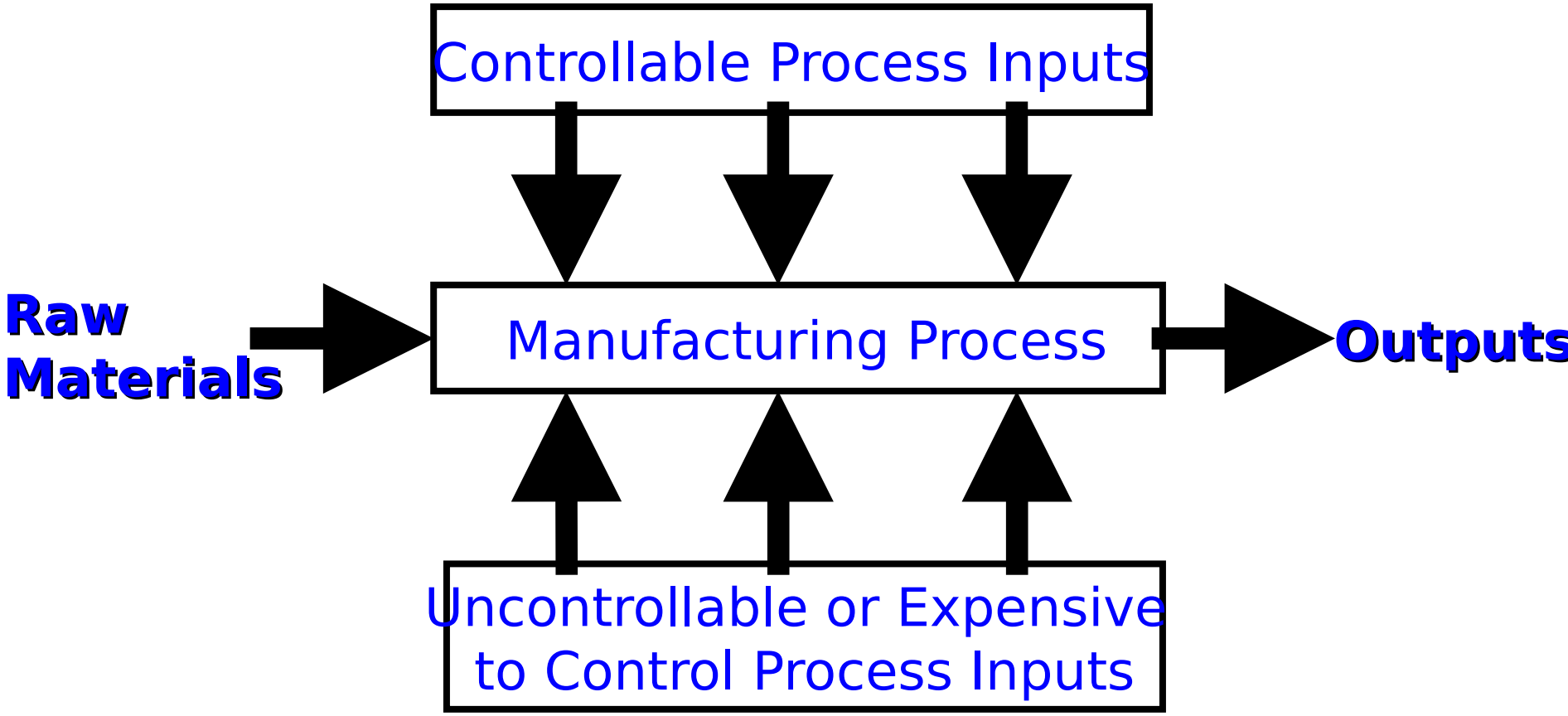
Theoretical Model
- -Physical principles understood
- -Predicts and extrapolates completely
- -Increased repeatability

Mechanical Model
- -Empirical
- -Use of DOE allows complete prediction within bounds
- -Increased physical understanding
- -Increased repeatability

Systematic Laboratory Collection of Data
- -Empirical
- -Increased predictive capability
- -Increased repeatability

Guess/Eyeball/Common Sense
- -Very limited predicative capability
- -Questionable repeatability
- "Heavy things fall faster"

The Typical Manufacturing Process



Why Experiment?

- Verify something that is believed to be true
- Investigate a hypothesis or “hunch”
- Determine the effect of using a method or material that has not been tried before
- Reduce costs, generate better operating results, etc

Methods of Analysis

- Best Guess or Experience
- One Factor at a Time (OFAT)
- All Combinations (full factorial)
- Orthogonal
  - Fractional Factorials
    - » **Classic**
    - » Taguchi
  - Box Behnken
  - D-Optimal
  - Others

What is a Designed Experiment?

- A method to determine how many tests and which test conditions are required to obtain a understanding of the effects of the factors and interactions.

- A formalized method of analyzing how factors, components or ingredients in a manufactured product affect its quality, performance, or other attributes.

Objective of Design of Experiments

- Maximum Information using Minimum of Resources
- Determine Influence of ***Factors*** upon the ***Response***
- Determine which combination of ***Factors*** and ***Levels*** Optimizes the ***Response***
- Identify ***Interactions***
- Build Empirical Models (Equations)

More DOE Objectives:

- Identify the important variables whether they be product or process parameters, materials or components from suppliers, environmental or measuring equipment factors.
- Reduce the variation on the important variables through close tolerancing, redesign, supplier process improvements, etc.
- Open up tolerances on the unimportant variables to reduce cost substantially.

Video #1

- Look for:

  -Controllable Factors
  -Uncontrollable Factors
  -Inputs to the process
  -Outputs
  -Manufacturing Processes
  -Sources of Variations
  -Are there interactions?

A Simple Case Study

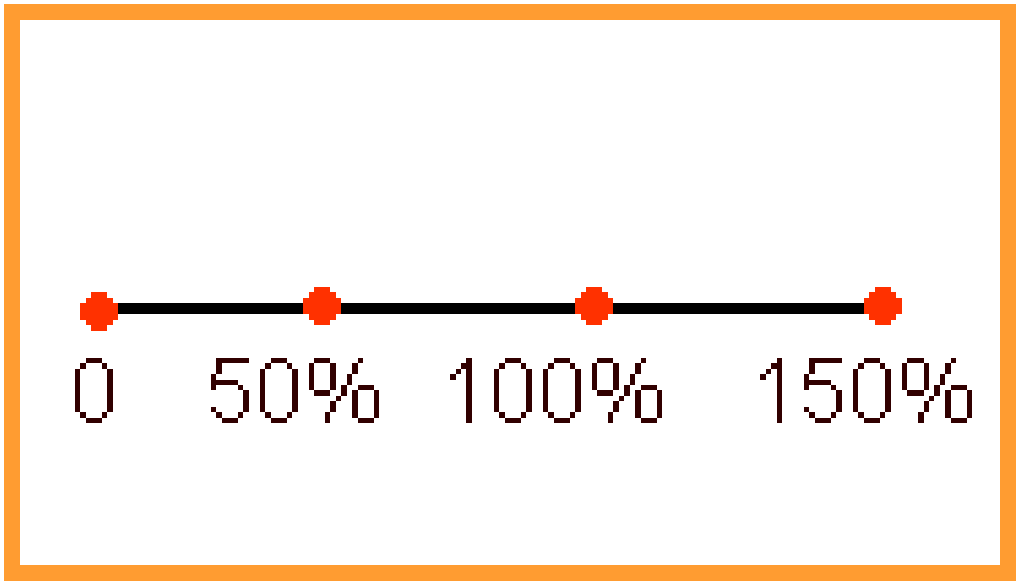
- The Chemical Department at Toxic, Inc. wants to study the effect of a particular herbicide soil pretreatment on the germination rate of soybean seedlings.
- Toxic decided to try 2 different application rates 4 weeks before planting :

  50% of recommended treatment rate

  150% of recommended treatment rate

1-D Experiment Design

- Suppose Toxic had decided to plant 100 seeds for each application rate and use the % that germinated as their ***Response*** measurement. The ***Design Space*** is one-dimensional regardless of the number of herbicide ***Treatments***



Experiment Design

- Others pointed out that the amount of rain that fell during germination could be significant. So, two different watering schedules were selected; dry and wet.

We now have 4 ***Treatments***; 2 X 2

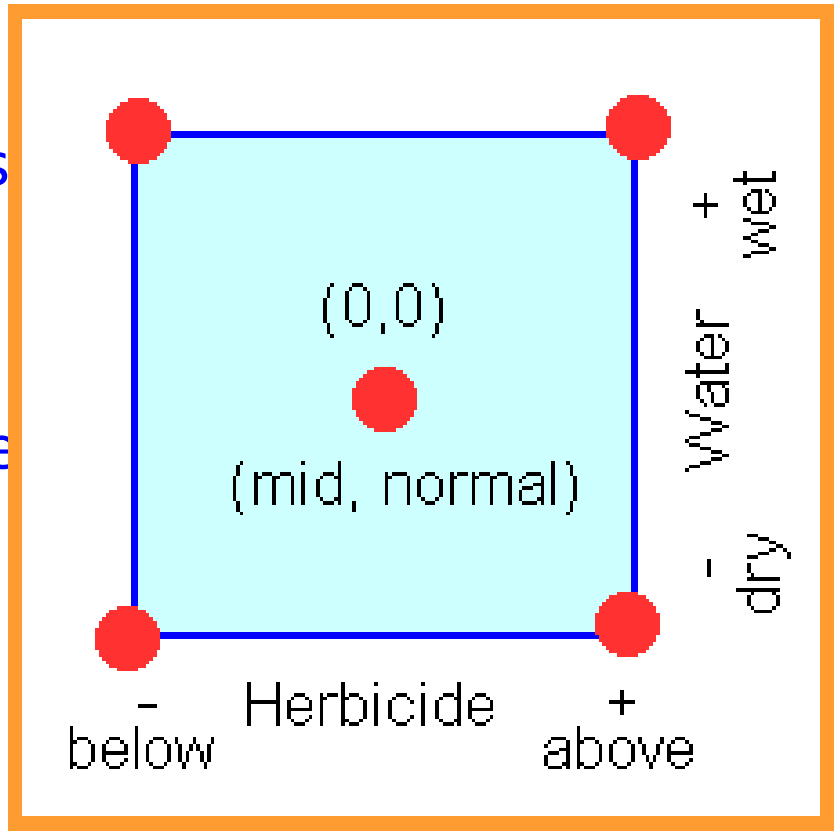
(Two levels of herbicide X two levels of rain)

The Design Space is now 2 Dimensional

2-D Experiment Design

The experimenters decided to look at only five combinations 2 herbicide amounts (50% over and 50% under) and 2 watering schedules (dry and wet) plus a normal or baseline condition.

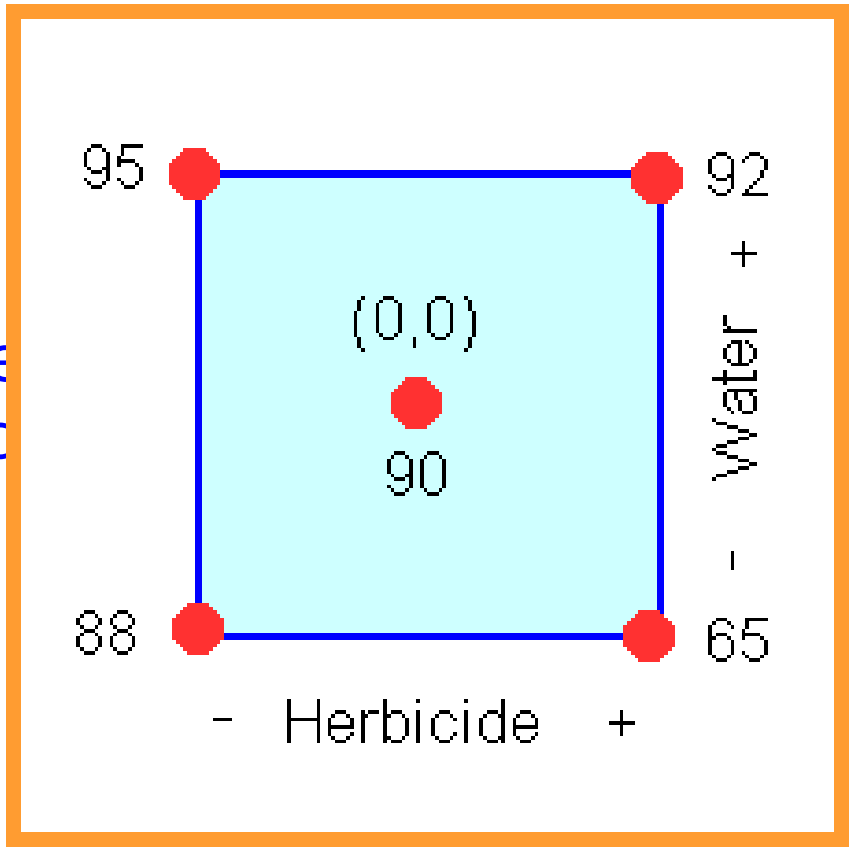
Now the design space is two dimensional.



The Results

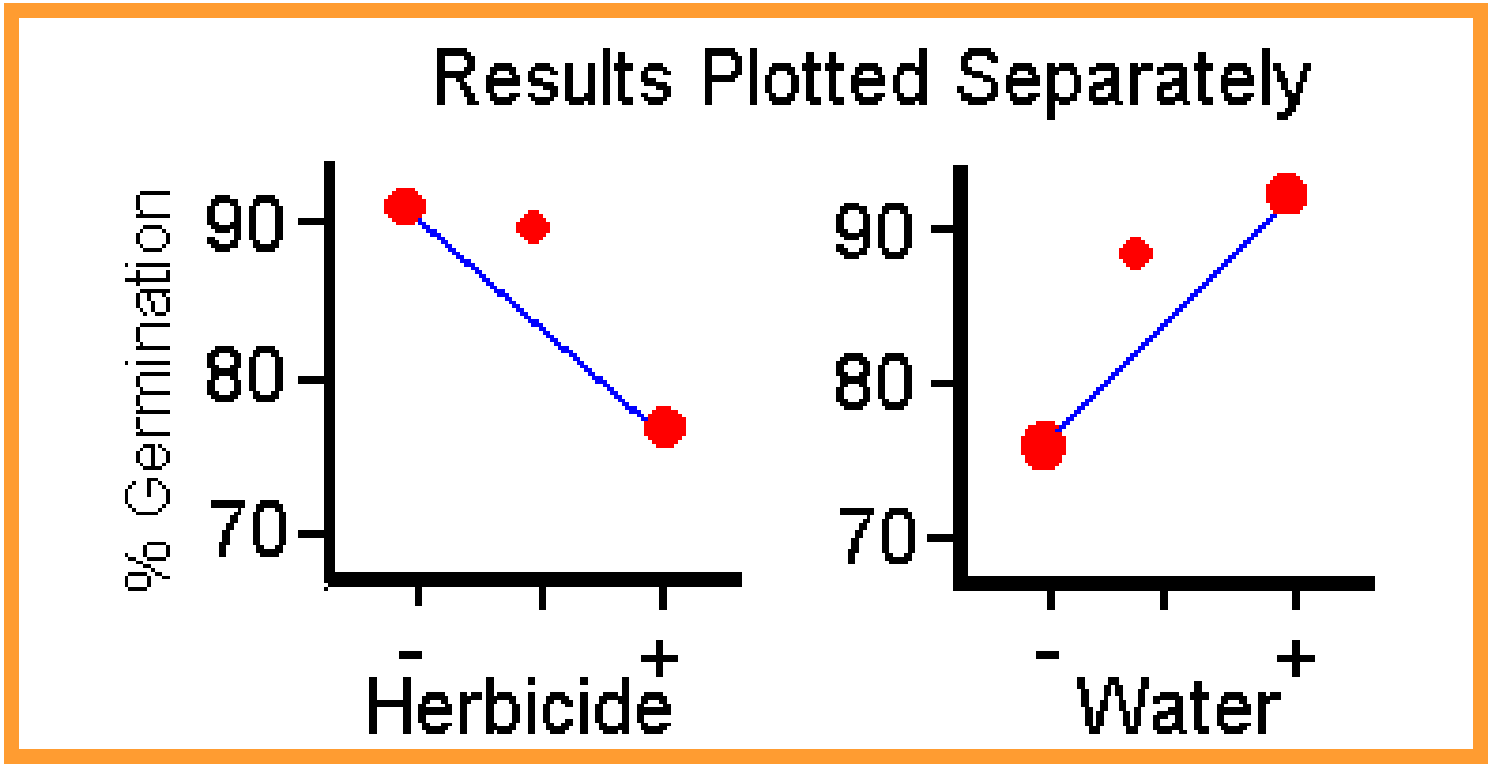
- After 4 weeks the germination rates are shown.
- It appears that herbicide level does have an effec

  as does the watering rate.

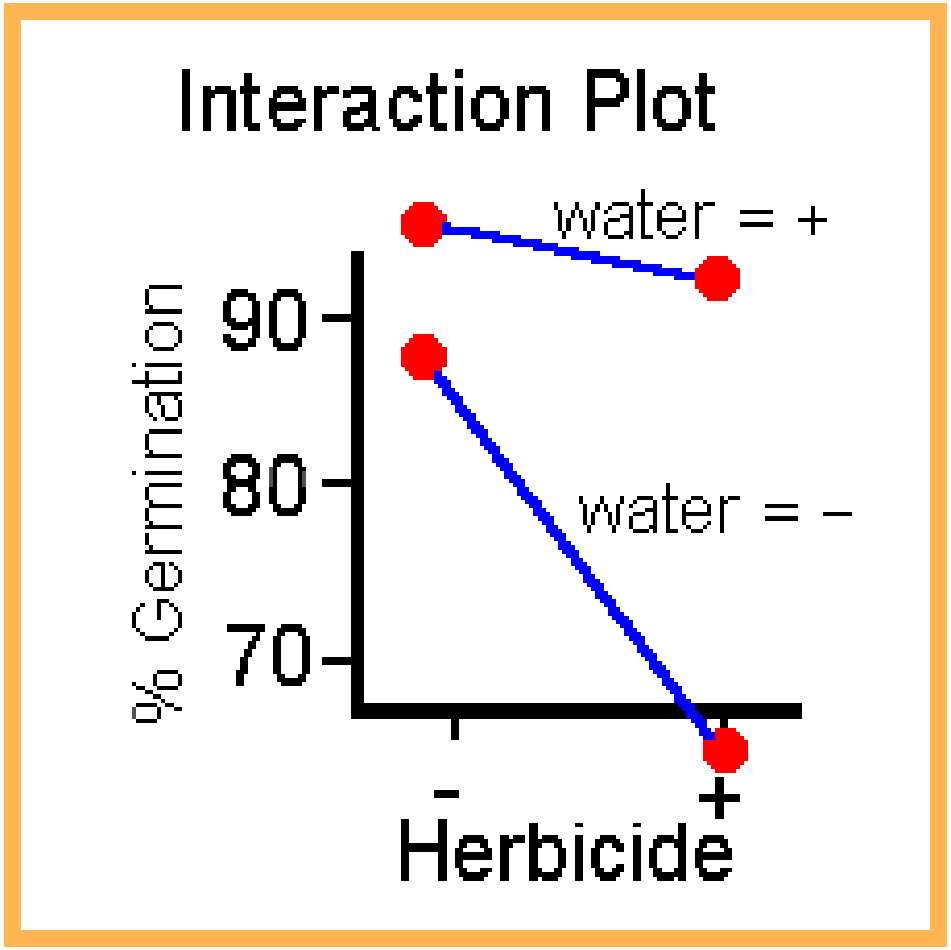


Experiment Results

- We now plot % germination vs herbicide and water in separate plots. What conclusions can we draw?



Experiment Results

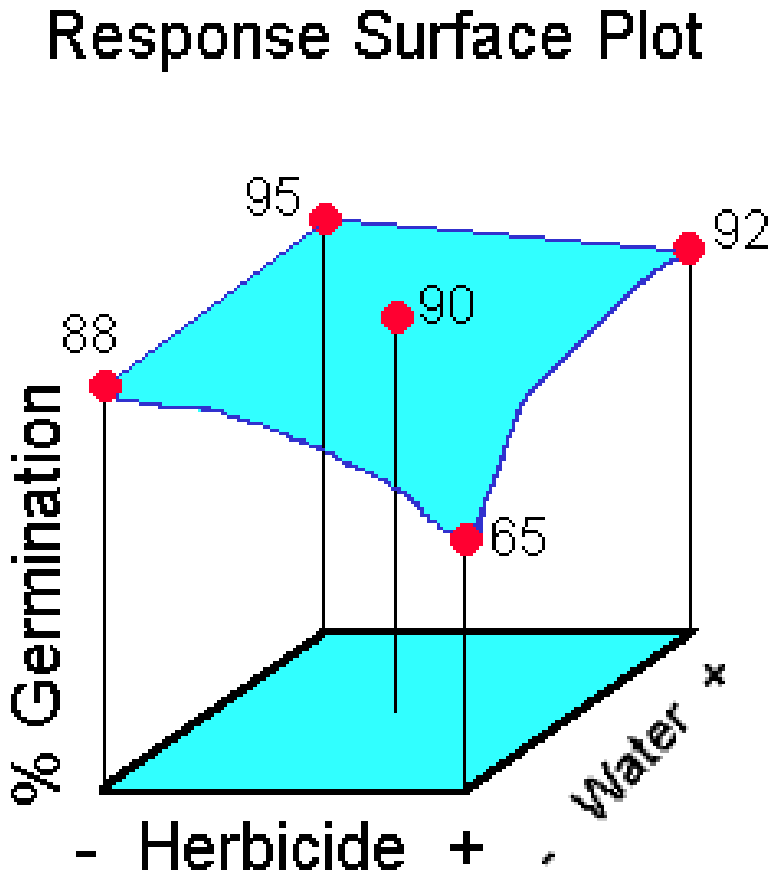


The lines are not parallel. This is an example of an ***Interaction*** - the joint effect of changing herbicide and water together is not what one would expect from when they are varied one at a time in isolation.

Experiment Results

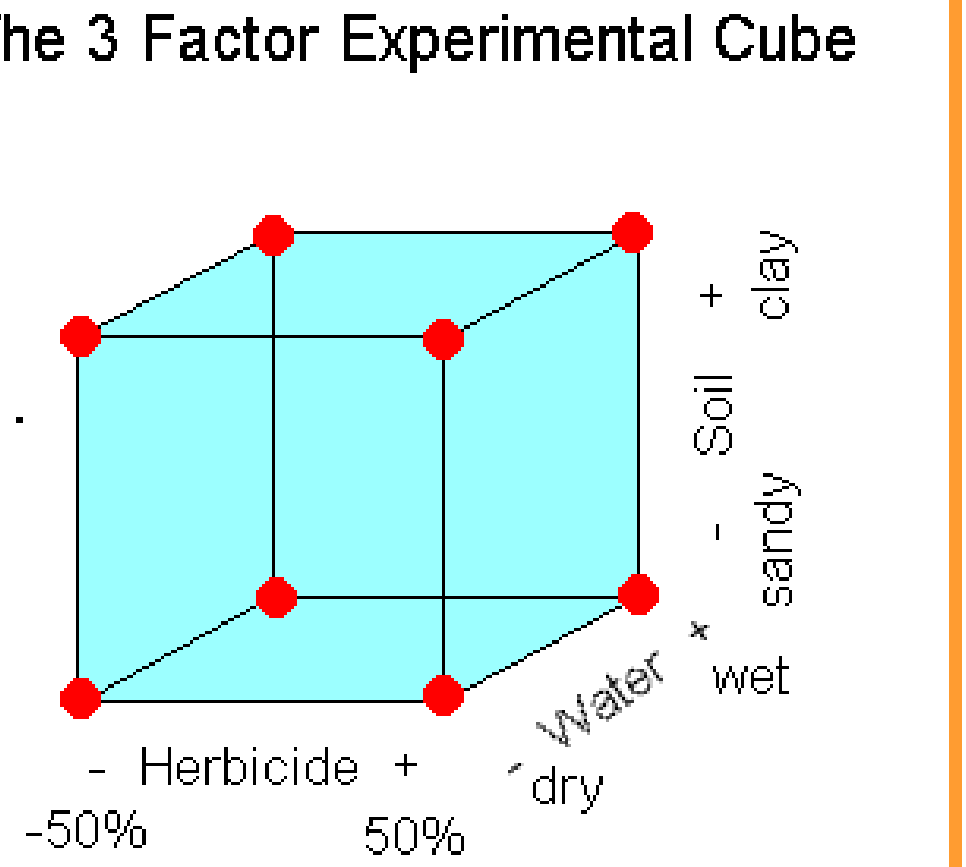
Plotted in three-dimensions is the ***response surface*** plot of the results.

=85+17(w)-13(h)-15(wh)



Experiment Results

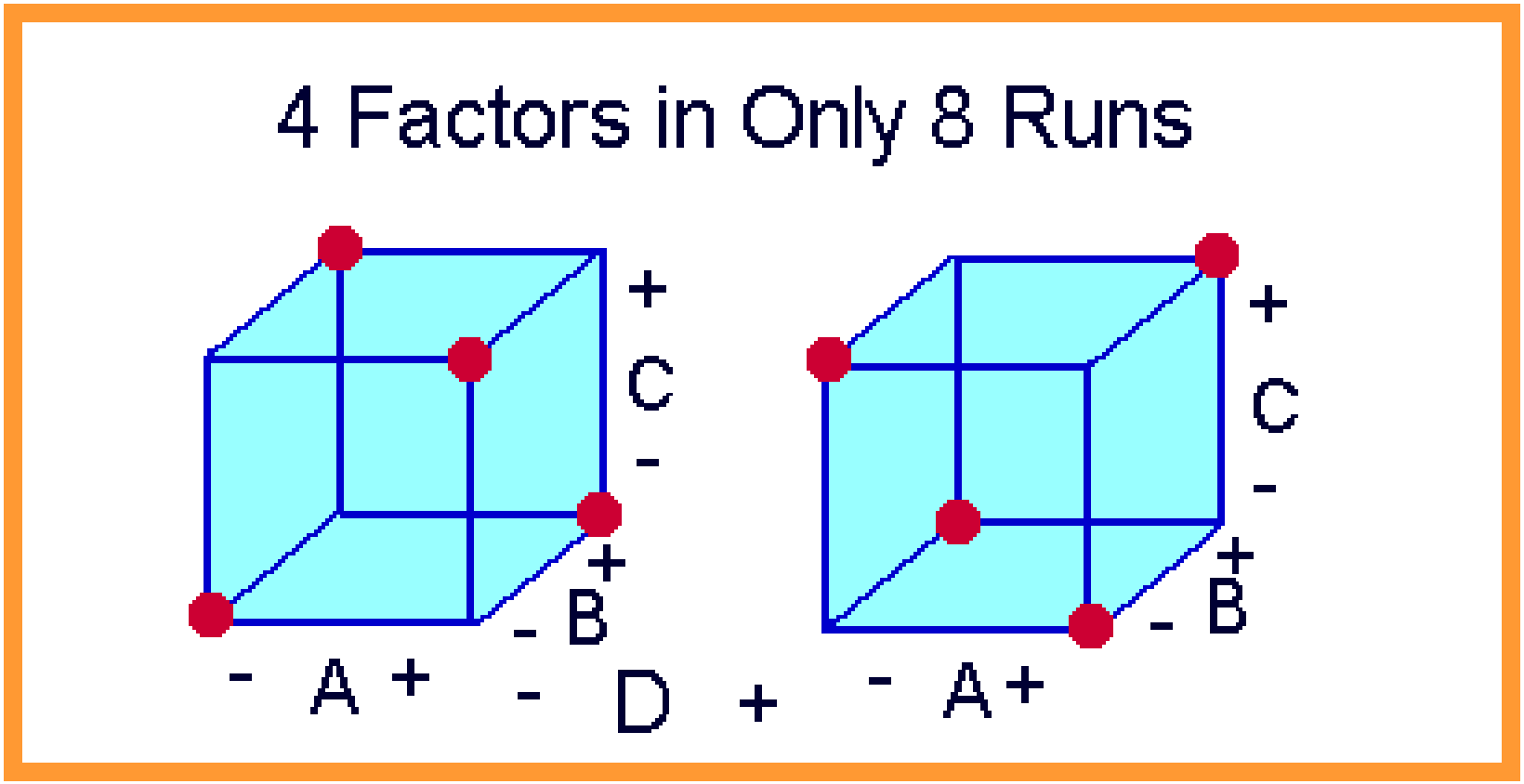
Another scientist then commented that they should have studied different types of soil. This adds another ***dimension*** to the experiment. The ***response*** (germinatic rate) would be the 4th dimension! We are nov working in Gaussian N space?!



Fractional Factorial Design of 4 Factors at 2 Levels

- We can get almost all the information we would get from a full factorial designed experiment, but only do half the number of experimental runs while maintaining all the main effects. This is called a ***fractional factorial design. See next slide.***
- Notice how the points define orthogonal surfaces of the cubes.
- Reduced the number of runs from 16 to 8 for a four factor experiment. In a 10-factor experiment, the number of runs are decreased from over 1000 to 64.
- More sophisticated models can analyze 15 factors is 16 runs! 2^15 = 32,768. **The statistics is in picking the right points! They are not randomly chosen!**
- But remember—we have now sacrificed the higher order interactions in a fractional factorial experiment! **You need process expertise to decide what level of interactions you wish to assume do not exist.**
- **We can only do this for "well-behaved" functions (processes).** We can interpolate but not extrapolate across the design space!

4-Factors Requires 8 Experiments!



Statapult Experiment

- What are the key factors?
- What factors are we ignoring
- What is the system response
- How can we characterize the performance of the syste as a function of these factors
- What position of these factor will give the best response?

Analytical Results

- 150° < arm angle < 180° or low = 150°, high = 180°. Call this factor "A" coded as +1/-1
- Tension hole (th) is low (-1) or high (+1). Call this factor "B"
- Rubber band hole (rh) is low (-1) or high (+1). Call this factor "C"
- Predictive equation can be developed as y=85.17+73.2A -32.3B+22.25C-21.08BC-15.33AB-11.75ABC+4.4AC
- Or we can analyze using the basic physics of the problem; which is intuitively obvious to the most casual of observers:

$$\frac{gI_o}{4r_B}\frac{(R+r_B\cos\theta_1)^2}{\cos^2(\frac{\pi}{2}-\theta_1)\left[r_B\sin\theta_1+(R+r_B\cos\theta_1)\tan(\frac{\theta}{2}-\theta_1)\right]}=\eta_c\int_{\theta_0}^{\theta_1}F(\theta)\sin\theta\cos\phi d\theta-(Mgr_G+m_gr_B)(\sin\theta_1-\sin\theta_0)$$